

Seminar 8

(S8.1) Let $A \in \mathcal{B}$.

(i) $A \setminus A_{ret}$ is wandering.

(ii) $A \setminus A_{inf} = A \cap \bigcup_{n \geq 0} T^{-n}(A \setminus A_{ret})$.

(S8.2) Let (X, \mathcal{B}, μ, T) be a MDS. If $A \in \mathcal{B}$ is such that $\mu(A) > 0$, then there exists $1 \leq N \leq \Phi$ such that

$$\mu(A \cap T^{-N}(A)) > 0,$$

where $\Phi = \left\lceil \frac{1}{\mu(A)} \right\rceil$.

(S8.3) Prove or disprove the following statement: "If $A \subseteq \mathbb{N}$ is a set of positive upper density, then there exist $x, y, z \in A$ such that $x + y = z$."